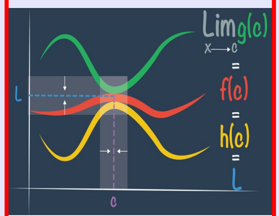
Calculus I

Lecture 36



Feb 19-8:47 AM

Given $f(x) = \sqrt[3]{x^2(6-x)}$

$f'(x) = \dfrac{4-x}{\sqrt[3]{x(6-x)^2}}$

$f''(x) = \dfrac{-8}{\sqrt[3]{x^4(6-x)^5}}$

Domain $(-\infty, \infty)$

Y-Int $(0,0)$

X-Ints $(0,0), (6,0)$

$f'(x) = 0 \to x = 4$

$f'(x)$ is und. $\to x = 0, 6$

C.N. $\to 0, 4, 6$

$f''(x) \neq 0$

$f''(x)$ is und. $\to x = 0, 6$

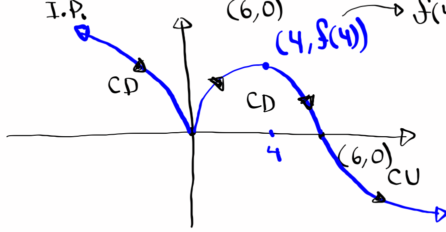
P.I.P. $\to 0, 6$

x	$-\infty$		0		4		6		∞
$f'(x)$		$-$	o	$+$	•	$-$	o	$-$	
$f''(x)$		$-$	o	$-$		$-$	o	$+$	
$f(x)$		Dec., CD		Inc., CD		Dec., CD		Dec. CU	

NO I.P.

I.P. (6,0)

$(4, f(4))$



$f(4) = \sqrt[3]{4^2(6-4)}$

$= \sqrt[3]{16 \cdot 2}$

$= \sqrt[3]{32}$

$= \sqrt[3]{2^5}$

$= \boxed{2^{5/3}}$

Apr 17-8:45 AM

$f(x) = \frac{x}{x^2+1}$

$f(-x) = \frac{-x}{(-x)^2+1} = \frac{-x}{x^2+1} = -\frac{x}{x^2+1}$

Domain $\to (-\infty, \infty)$

No V.A.

$\lim_{x \to \pm\infty} f(x) = 0$

H.A. at $y=0$

Y-Int $(0,0)$

x-Int $(0,0)$

$f(-x) = -f(x)$

$f(x)$ is odd $\to$ Symmetric w/r origin

$f'(x) = \frac{1(x^2+1) - x(2x)}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$

$f'(x) = \frac{(1-x)(1+x)}{(x^2+1)^2}$ C.N. $\to x = \pm 1$

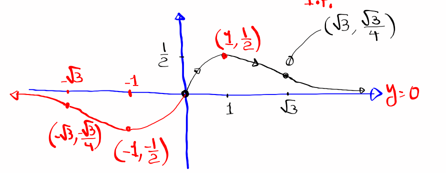
$f''(x) = \frac{-2x(x^2+1)^2 - (1-x^2)\cdot 2(x^2+1)\cdot 2x}{(x^2+1)^4} = \frac{\cancel{(x^2+1)}[-2x(x^2+1) - 4x(1-x^2)]}{(x^2+1)^4}$

$= \frac{-2x^3 - 2x - 4x + 4x^3}{(x^2+1)^3} = \frac{2x^3 - 6x}{(x^2+1)^3} = \frac{2x(x^2-3)}{(x^2+1)^3}$

$f''(x) = \frac{2x(x-\sqrt{3})(x+\sqrt{3})}{(x^2+1)^3}$ P.I.P. $\to 0, \pm\sqrt{3}$

x	$-\infty$ $-\sqrt{3}$ -1	0		1		$\sqrt{3}$		∞
$f'(x)$			+		−		−	
$f''(x)$			−		−		+	
$f(x)$			Inc. CD		Dec CD		Dec CU	

I.P. $\to (\sqrt{3}, \frac{\sqrt{3}}{4})$



Apr 17-9:02 AM

The sum of two positive numbers is 16.

$x>0$, $y>0$ $x+y=16$

What is the smallest possible value of the sum of their squares? $x=1, y=15$

$x^2 + y^2 \Rightarrow x^2 + (16-x)^2$

$x^2+y^2 = 1^2 + 15^2 = 1 + 225 = 226$

Let $f(x) = x^2 + (16-x)^2$

$f'(x) = 2x + 2(16-x)^1 \cdot (-1)$

$= 2x - 2(16-x) = 2x - 32 + 2x = 4x - 32$

Max or Min $\to$ C.N. $\to 4x - 32 = 0 \to x = 8$

$f''(x) = 4 > 0 \to$ C.U.

$(8,8)$

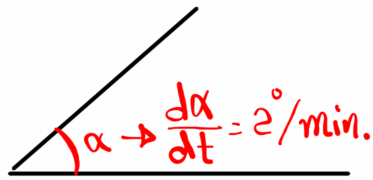
$x^2+y^2 = 8^2+8^2 = \boxed{128}$ Min. Sum of their Squares

What if $x=6, y=10$

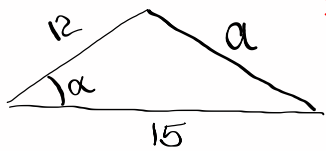
$6^2 + 10^2 = 136$

Apr 17-9:20 AM

Two sides of a triangle are 12m & 15m. angle between them is increasing at a rate of $2°/\text{min}$.



How fast the third side is increasing when the angle is $60°$?



$\frac{da}{dt}$ when $\alpha = 60°$

Review Law of Cosines and finish this Problem.

Apr 17-9:28 AM

Class QZ 18

Verify that $f(x) = \frac{1}{x}$ satisfies the conditions of MVT over $[\frac{1}{4}, 1]$, then find all numbers c that satisfy the conclusion of MVT.

$f(x) \to$ Domain $\to x \neq 0$

$f(x)$ is Cont. on $[\frac{1}{4}, 1]$

$f'(x)$ is diff. on $(\frac{1}{4}, 1)$

$f(1) = 1$, $f(\frac{1}{4}) = 4$

$f'(x) = \frac{-1}{x^2}$

$f'(c) = \frac{f(b) - f(a)}{b - a}$

$\frac{-1}{c^2} = \frac{1-4}{1-\frac{1}{4}}$ $\quad \frac{-1}{c^2} = \frac{-3}{\frac{3}{4}} \to \frac{-1}{c^2} = 4$

$-4c^2 = -1 \to c^2 = \frac{1}{4}$ $\quad c = \pm\sqrt{\frac{1}{4}}$ $\quad c = \pm\frac{1}{2}$

$\boxed{c = \frac{1}{2} \text{ on } (\frac{1}{4}, 1)}$

Apr 17-9:35 AM